How to Use Black Holes as Particle Detectors

Ian Lim

May 16, 2017

Question:
Do massive photons exist?

Also, who cares?

Some Motivation

- The photon is the particle of light. It mediates the electromagnetic force.
- Physicists believe the photon is massless. This property is what lets light travel at the speed of light.
- But what if the photon (or a photon-like particle) has mass?
- How would we know?

Roadmap

- Black holes and no-hair theorems
- From classical electrodynamics to massive electrodynamics
- Putting it together: black hole discharge for a massive photon
- Future work

Introducing black holes

- Astrophysical phenomena popularized by science fiction (Interstellar, Doctor Who, and others)
- What are they? “Point mass” solutions to Einstein’s equations of general relativity, like point charges in E&M

…and "no-hair theorems"

- Popularized by John Archibald Wheeler ("Black holes have no hair")
- Black holes are very simple! Just need three classical parameters:
  - Mass
  - Charge
  - Angular momentum

Classical Electrodynamics

- In regular classical electrodynamics, all of electricity and magnetism is governed by a famous set of equations called the Maxwell equations.

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Sometimes we write them as four equations relating the electric field *E*, the magnetic field *B*, the current density *J*, and the charge density ρ.

$$\partial_\nu F^{\mu\nu} = \mu_0 J^\mu$$

Other times, we rewrite these equations more compactly using “index notation.”

~~Classical~~ Massive Electrodynamics

- What happens when we turn the photon mass on?

$$\partial_\nu(\sqrt{-g}g^{\nu\alpha}g^{\mu\beta}F_{\alpha\beta}) = 4\pi\sqrt{-g}J^\mu,$$

- This is the equation for the electric field around charged stars and black holes in massive electrodynamics (*m* is the photon mass)
- *F* and *A* contain information about the electric and magnetic fields, *J* contains any sources like electric charge and current, and *g* describes the curvature of space and time.

Solving Maxwell-Proca

- For simplicity, consider the spherically symmetric case. Are there static charged solutions?
- No! Turns out an observable (the scalar product of the vector potential) diverges at the horizon– not physical behavior! So what does this mean?

In massive electrodynamics,
black holes must discharge!

How does it happen?

- Remember the correction term? The massive field corresponds to a current proportional to the photon mass squared!

$$J^\mu \sim m^2 A^\mu$$

- This means that a “screening charge” carried by the field itself falls into the black hole to cancel out the charge that initially fell in.
- The rate of change *dq*/*dt* suggests exponentially fast discharge– by dimensional analysis, it happens at a rate γ proportional to $m^2 r_g$.

The small-mass limit

- In the limit of small photon mass, we have a simple fitting formula for the decay rate as a function of *m* and r_g.

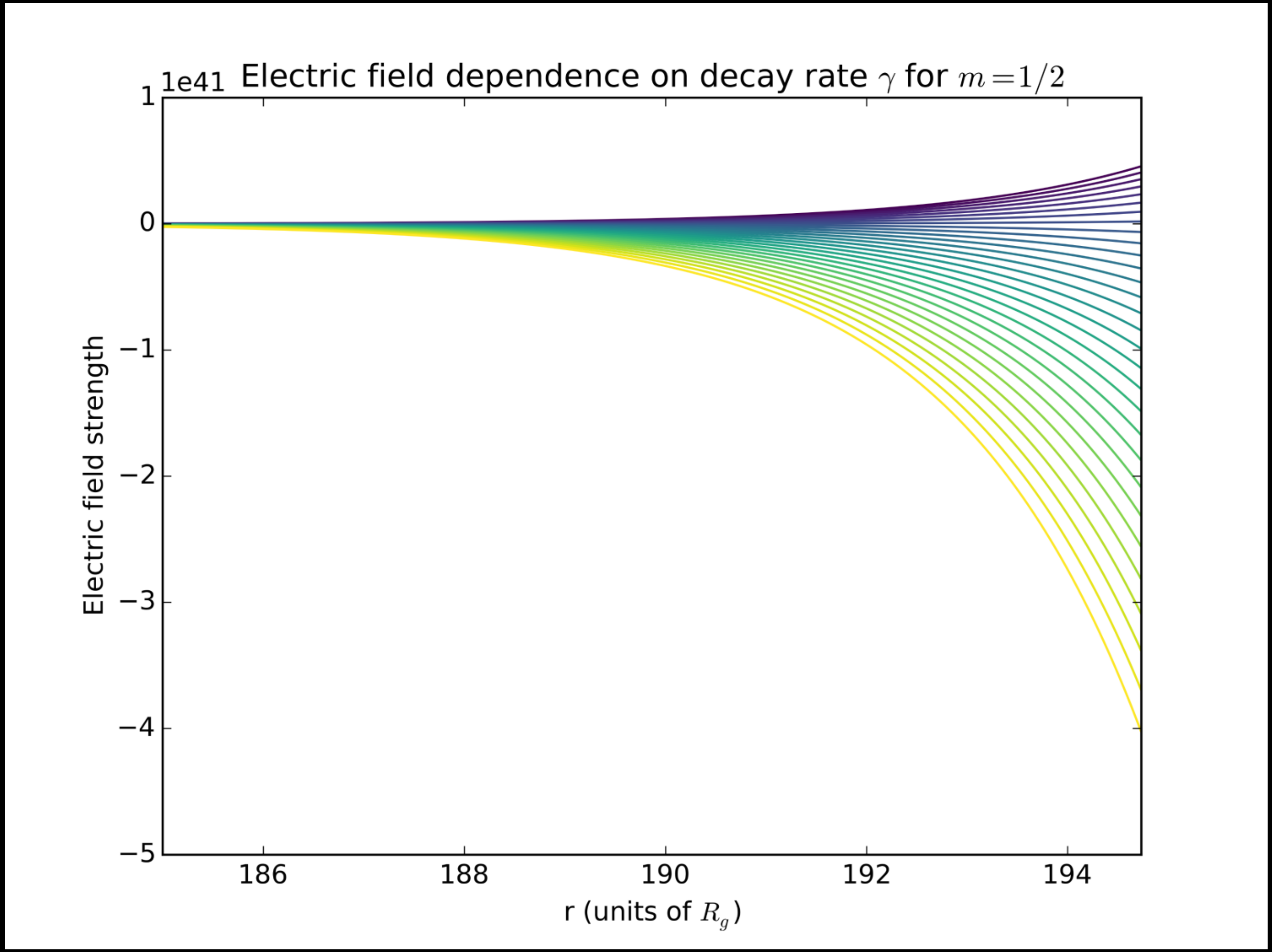
$$\gamma \approx \frac{m^2 r_g}{1 + m r_g}.$$

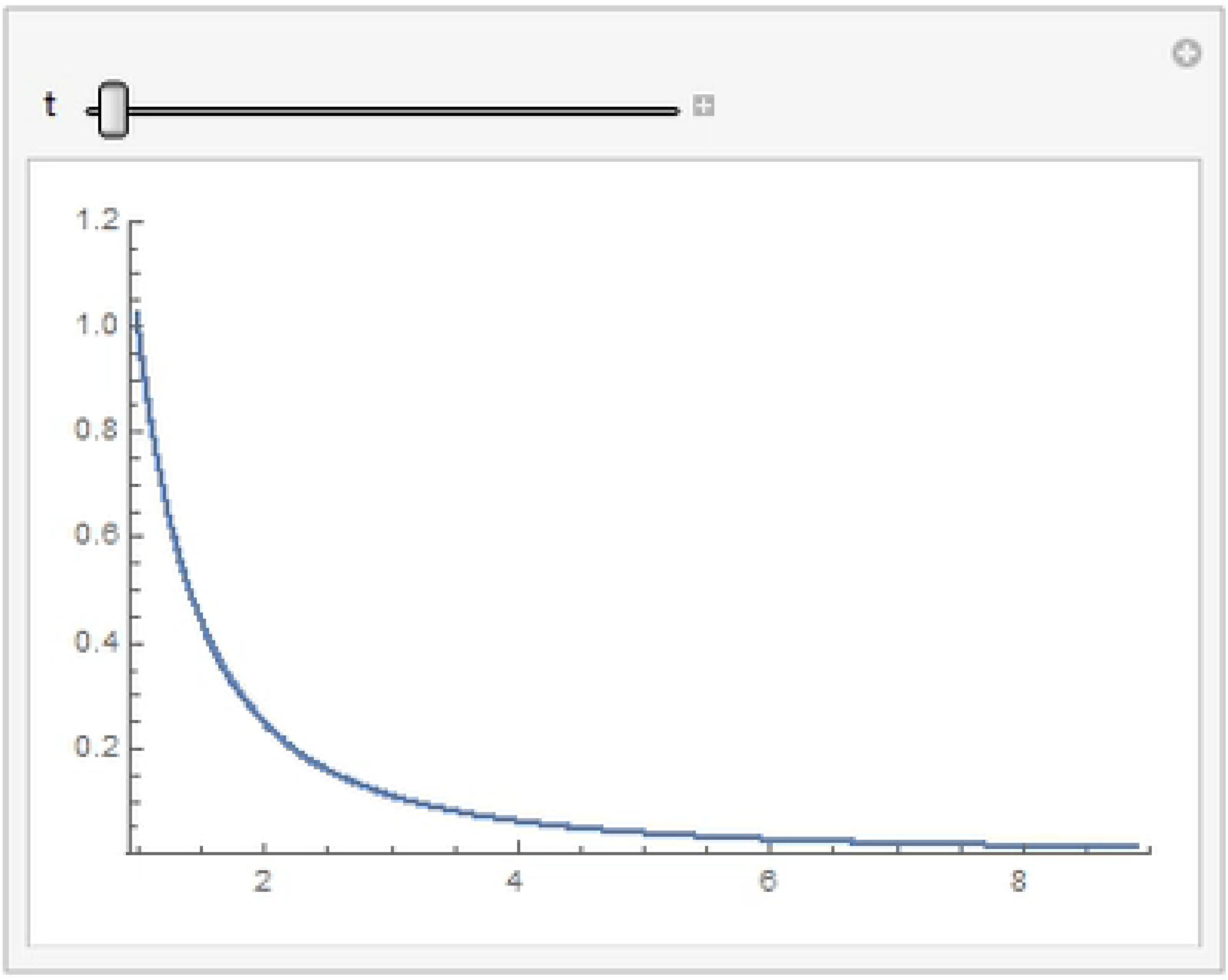
- This makes for a good starting guess in the numerical integration.

How fast does it happen?

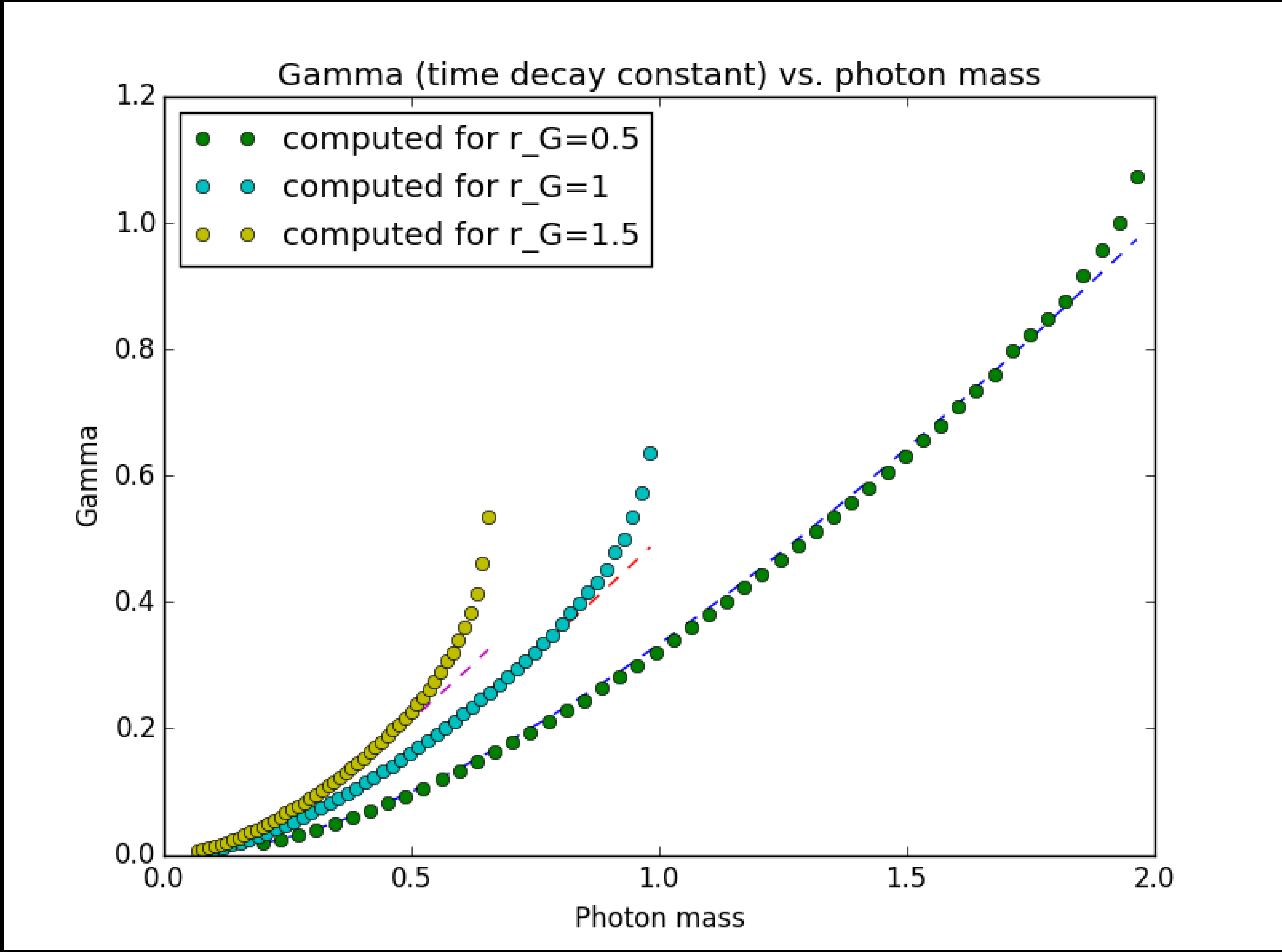
- By making a smart guess about the time dependence of the electric field, we turn the Proca equation into something we can solve numerically!
- To recap, there are two physical parameters we can tune:
  - m, the photon mass
  - r_g, the Schwarzschild radius
- And one parameter we introduced:
  - γ, the time decay rate of the field
- But for every choice of m and r_g, only one decay rate gives a physical solution.

Some numerical solutions





t
1.2
1.0
0.8
0.6
0.4
0.2
2
4
6
8



Gamma (time decay constant) vs. photon mass
computed for r_G=0.5
computed for r_G=1
computed for r_G=1.5
Gamma
Photon mass
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0.0
0.5
1.0
1.5
2.0

What happens in the large mass limit?

- We have confirmed the fitting formula to within 3% accuracy for $mr_g < 0.5$, but for larger m this formula seems to break down.
- Preliminary calculations suggest that the decay rate blows up as m approaches $1/r_g$.
  - This may be a quirk of numerics, or an indication that our guess for the decay rate fails sooner than expected.
- Future work will focus on finding general solutions to the Proca equation without assuming exponential time decay.

What happens in the large mass limit?

- Finding a more general solution will help us understand what this discharge process would look like.
- In the real world, charged black holes are rare but might be produced briefly by binary mergers. The gravitational waves from this merger would be altered by the discharge process during the ringdown.
- In this way, we can use the gravitational wave signals from black hole mergers to look for echoes of massive photon discharge.

To recap:

- The Proca equation is a modified version of the Maxwell equations for massive electrodynamics.
- There are no static charged solutions to the Proca equation (i.e. with nontrivial field profile) so charged black holes must discharge!
- For small photon mass, the electric field is well-described by an exponential time decay. But as the photon mass grows larger, this assumption may break down.
- More work is needed to understand this limit. Either we will extend the decay solution to larger mass or find a more general expression for time dependence.

Acknowledgements

- Thanks to Savas Dimopoulos for suggesting and advising this project
- Junwu Curly Huang and Anson Hook, for being constant points of contact and sounding boards
- Peter Graham, for being my second reader
- Sean Hartnoll, as my major advisor in the physics department
- Mehrdad Mirbabayi and Andrei Gruzinov, for their analysis of the discharge process
- My parents for their constant love and support. ☺

Thank you for your time!

Any questions?

- Where does the charge go?
  - A screening charge of opposite sign corresponding to the field itself moves into the black hole to cancel out the charge that initially fell in.
- What are the units?
  - We work in Planck units with $c=G=1$. r is in units of the Schwarzschild radius and m is in units of the Planck mass (so mR_g is dimensionless and m^2R_g has units of 1/time)